Pearson Edexcel

Examiners' Report
Principal Examiner Feedback

January 2024

Pearson Edexcel International Advanced Level in Further Pure Mathematics F1 (WFM01)
Paper 01

WFM01 January 2024 Examiner report

General

Generally, the paper was well received with many excellent responses seen, though there were a couple of discriminating areas. Most of the students answered the questions fully with no evidence of issue of timing being apparent, most students completing to the end of question 10. There were good algebra skills on display, though presentation of solutions was sometimes poor in terms of demonstrating the full workings of solutions, especially in the likes of question 6, where it seems students rely heavily on their calculators. The importance of demonstrating method must be made clear so that marks may be gained even when answers are incorrect if the working shown was correct.

Individual Question report

Question 1

An accessible start to the paper, with the work on the determinant and inverse expected and well understood topics, though some were not able to fully reason in part (a) why it was necessarily non-singular.

Calculating the determinant of **M** presented no problems for most, but many students missed out on the marks for showing that det **M** $\neq 0$ by simply stating that $k^2 + 2k + 4 \neq 0$ which was insufficient. There were a few who failed to obtain a suitable determinant by only considering the leading diagonal terms, but these were rare, while sign errors in the determinant were equally rare. Slips in simplification did occur, but these were still able to access the second method.

Some found the discriminant or solved the quadratic to show the roots were not real and some attempted to complete the square with a surprising number of errors stating $(k + 2)^2 + \ldots$ rather than the correct expression of $(k + 1)^2 + 3$. Those who obtained correct expressions or roots usually scored full marks, though some students failed to secure the final mark for part (a) by omitting a conclusion that **M** was non-singular after showing that det **M** $\neq 0$.

A small number of students did not know what non-singular meant, so left this part out.

The marks for part (b) were scored by most students, with the method for the inverse being well versed. Few slips were made in the adjoint, a sign error in the $-k - 7$ term being the most common such, with the division by the determinant being shown by the vast majority. There was no need to incorporate the 1/det **M** inside the matrix by multiplying out each element, but many students were taking no chances and gave the version with 1/det **M** left as a factor outside the bracket and the version with each element with $k^2 + 2k + 4$ (or equivalent) as the denominator. A few students only gained M1A0 as they had an incorrect determinant, while others recalculated the determinant unnecessarily.

Question 2

Again, this question proved highly accessible, with most able to make progress to at least part (c), occasional slips in algebra being the main culprit of loss of marks.

Almost every, if not every, candidate gained the B mark by correctly identifying 5+4i as another root in part (a).

For part (b) some students went straight from the two complex roots to the required quadratic by simply identifying the sum and product of roots. Most students achieved the correct quadratic factor and proceeded via the 'main' method, although a surprising number of students used long division rather than factorisation by inspection. Others used the sum and product of roots of a cubic to find the third root efficiently. A small number having correctly identified $2z - 1$ as a factor then failed to give the 3rd root as $z = \frac{1}{2}$ with some only identifying it in part (d), too late to score the mark, and some not including it at all in their solution.

Long division was quite commonly used to answer part (c) also, and mistakes here were frequent. Slips in arithmetic in determining the values of p and q were fairly frequent, and often these values were found before deducing the third root, by using that the remainder of the long division must be zero. Use of the factor theorem to first find p and q before deducing the root was also observed, with students not always spotting the most efficient methods. That was surprising as this is a very well tried and tested type of question where the method expected of the main scheme ought to be well known.

Part (c) was more discriminating, testing students' understanding of the geometry of the roots. The area of the triangle was usually found correctly by students who had identified $z = \frac{1}{2}$ as the third root, and by the correct method for students with other values, most often $z = -\frac{1}{2}$, though some gave the answer 18i, gaining no credit. But many used the origin rather than the correct point, and so gained neither method here. In some cases students simply drew the Argand diagram.

Question 3

Most students had little difficulty with this question, with many fully correct responses seen. To find the required points, the equation of the tangent was needed and this first required them to find $\frac{dy}{dx}$ in terms of t. Those who left their $\frac{dy}{dx}$ in terms of x and/or y could not gain any marks (unless recovered by later substitution to imply the correct gradient). But most realised what they needed. Some seemed to have memorised the formula for gradient of a general point on a hyperbola, which was acceptable as a calculus method was not specified in this instance. But most attempted a differentiation method in order to obtain the gradient, with various approaches seen. A few incorrectly differentiated $y = \frac{c^2}{x}$ as $\frac{-2c}{x^2}$ (not treating *c* as a constant) or made slips in parametric differentiation but most attempts were correct. Those who achieved $\frac{dy}{dx} = -\frac{1}{t^2}$ usually went on to secure full marks. Errors at the start led to complicated expressions involving c and *t* and a subsequent failure to score the final few marks. Incorrect values often led to expressions for the area which when equated to 90 could not be solved for *c*. A small number of students confused the two coordinates, and mislabeled but had the correct two sets to be able to score in (b). Some students used the $y = mx + c$ and finding *c* approach. They did not always distinguish between this *c* and the *c* given in the question.

Those who did find *A* and *B* correctly had few problems in part (b) in using the given information to construct an equation to find *c*; however some gave 2 values for *c* and failed to notice the given "*c* is a positive constant". A careful reading of the question is advised, as this error led to the loss of the final mark.

Question 4

Though largely well answered, the language needed to describe the transformation was often lacking, and the transformation in part (a) was often poorly described, with a significant number of students not recognising a stretch or not using the correct terminology. A common mistake was to use "sketch" instead of "stretch", but words such as "strength" or misspellings such as "stetch" were also seen. Candidates were more likely to give a '*y* direction – factor of 3' type of answer than to state 'stretch'. 'Enlargement' was a common description, along with various other vague indications such as "times all *y* coordinates by 3". There were also numerous students who added "about *O*" to their description, which is meaningless in this case. Stretches appear to be less well understood as transformations than rotations or reflections.

The rest of the question was generally answered without much difficulty although the order of multiplying the matrices for part (c) was sometimes the wrong way round. Some students failed to apply the result given in the formula booklet. There were sometimes errors in finding the scale factor using the determinant of **C**. A few realised that det **B** = 1 as it is a rotation about *O*, and that the scale factor for **C** was therefore 1 x 3 without needing the determinant of **C** at all. Relatively few applied the scale factor incorrectly, obtaining $\frac{5}{3}$.

Question 5

This was another expected and fairly routine question, with algebraic slips accounting for the majority of the marks lost. The identities were dealt with well throughout, and the theory well understood.

The mark in part (a) was almost always achieved, and there were only very few examples of students calculating the roots directly, which was not permitted by the question.

Part (b) was also almost always successfully done with only occasional sign errors. Substitution into the correct form was normally successful even if errors in signs did creep in. A few students worked with decimals which sometimes complicated later parts.

In part (c) a few had sign errors when expanding the product of roots, but manipulation of the required expressions was otherwise generally fine, only a few marking errors. One reasonably common error was to see students attempt to multiply through by either $\alpha\beta$ or $(\alpha\beta)^2$, in order to make the calculations easier, but invariably forfeiting marks for using wrong expressions. Evaluations of the expressions were less than perfect on occasion, for instance swapping the calculated values of $\frac{3}{2}$ and $\frac{7}{2}$ was not uncommon, or just plain miscalculations from correct identities with substitution shown were made. Nevertheless the majority were able to successfully find the values needed.

The method for finding the new equation was very familiar territory, with most able to score the final method from their values for the sum and product. One or two did mix up the position of the coefficients of the final equation, while only a small number omitted the "… = 0"). A fairly surprisingly common error seemed to be omission of the *x* in the final equation, resulting in loss of the final two marks if it were not present in an unsimplified form initially.

Question 6

Numerical methods questions usually provide a good source of marks for students on WFM01 and this was no exception. Many responses were fully correct or lacked just an accuracy mark. Linear interpolation provides the biggest challenge where methods were lost.

In part (i)(a) most students knew the correct method but lost a mark because they did not complete the required information, usually by missing out "continuity" from the conclusion, though this was much less the case than in previous series. Very few students worked entirely in degrees, though it was clear many started in degrees mode before releasing this did not yield a change in sign, and so switched to radians mode. But most were able to calculate the two values correctly and identify the sign change.

For part (i)(b) the main error was using a negative distance for one of the terms in the interpolation. Those who drew a small sketch tended to be more successful with the question as were those who had been taught to use the modulus of the distance. A few tried to use the interval bisector method whilst others just left this part out. There were a few instances of students quoting an already rearranged formula, or of giving just an answer with no working, presumably from extensive calculator use. But a method should be shown to ensure full marks will be gained.

There was greater success in part (ii) with the majority correctly differentiating (which for a simple polynomial should be expected at Further Maths level), with just a few unable to deal with the negative index correctly. Slips in coefficients were rare. In part (b) the Newton-Raphson process was very well completed with few errors seen, and usually the method gained even following incorrect derivatives. However, again here, some were reluctant to show the method and simply gave an answer.

Question 7

The second of the coordinate geometry questions on the paper, this did prove the more challenging of the two, though again was in the main very well answered.

For this question a calculus method was required in part (a) so suitable working was required to be shown. Those who had memorised the derivative for a parabola could therefore only access one mark. However, most did show a method, and this was generally well completed. There were a variety of approaches – implicit differentiation , forming y =... and directly differentiating, and parametric differentiation all common – with problems arising from putting the first two in terms of t. As with question 3, some failed to write their derivative in terms of t at all, and quoted the given equation from incorrect working. Such were unable to access any marks in this part. A few attempted the tangent instead of the normal, yet still managed to end up at the given equation. But generally the method for the normal was done well, via either $y - y_1 = m(x - x_1)$ or use of $y = mx + c$ in order to find c.

In part (b) the majority of students were able to access the first mark, giving the correct equation for the normal at $t = 9$, although a couple forgot to substitute for t. Again, the majority then knew the theory of how to proceed although some errors were seen. Most proceeded via an equation y with only a few using the x or t approaches. Common errors included getting confused with the algebra, (it was easier to write the equation in terms of y), miswriting the order of the coordinates (which lost the last mark unless x and y had been specified beforehand) and of finding x using the quadratic equation rather than the line and thus having the wrong sign for y. Some wrote the equation in y and solved it and then repeated the exercise for x and some used decimals, rather than keeping exact. Additional coordinates with the signs varied were also given by a few students.

Once again in this question an over reliance on the calculator was observed with some showing no working for the produced solutions following correct statements of the two equations to solve, despite the fact that acceptable calculators should not be able to simultaneously solve a quadratic and linear equation.

Question 8

Another very accessible question, especially this late in the paper. Summations are another expected and well handled topic and there was nothing unusual in this question to cause a particular challenge. The ability to write out a proof does nevertheless provide a challenge to some with the given answer for part (a) appearing from insufficient steps relatively frequently.

The vast majority of students were able to expand the initial brackets and apply correct identities in part (a), scoring the first two marks for a fully correct expression. A few had an incorrect final term, usually "n". Most students with correct expressions went on to correctly factorise by $n(n+1)$ but a few then were careless with signs, and did not always simplify to a three term quadratic before giving the answer. However, some instead expanded to a quartic, and these usually then stated the final, given, answer without supporting working to justify the final two marks.

Only a small number attempted induction for this part, with no marks available for this.

Though some did not attempt part (b), most again knew the correct method and proceeded to score at least the first mark, albeit in some cases by subtracting f(n) rather than f($n-1$) from f($n+1$). In this latter case no further marks were possible.

While most with a successful first step attempted to factor out the n and $n-1$ terms, again some students were clearly reliant on their calculators to factorise the quadratic in the final part of the question as sometimes evidenced by the factor $\left(n+\frac{4}{15}\right)$ in their final answer rather than $(15n+4)$, though often the correct factored form was given following a quartic with no intermediate working. Those who made genuine attempts to factorise sometimes lost the final two marks for losing track of one of the coefficients of 2, and obtained an incorrect quadratic factor.

Question 9

This was less well completed than earlier questions, placing it well at the tail end of the paper. The final mark of the question proved particularly discriminating. Very few scripts were seen that gained full credit for this question.

In part (a) most were able to make some progress, though a few did not manage to get over the first hurdle, not for lack of trying. There were some very roundabout answers to this part. Of the more successful attempts all 3 approaches highlighted on the mark scheme were seen, with the main scheme the most common, but Way 3 very rare. Most exhibited the procedure of multiplying numerator and denominator by the complex conjugate of the denominator at some stage in the working, and most were able to deal with the i^2 terms, though this was slightly less successfully completed. Those who navigated the first two steps of the method almost invariably proceeded to make *z* the subjecting meaning the majority of students scored the three method marks. However the accuracy mark was much less accessible, with algebraic errors being common, losing a square on the λ in the denominator being an example of a common error. A few lost the λ^2·entirely, while similar errors in the numerator were also made. Some students worked correctly but did not separate out the real and imaginary parts sufficiently to score the final mark. The question did not ask for simplest form which was fortunate for many students who seemed unable or unwilling to eliminate obvious common factors with $\frac{9\lambda^2 - 72}{9\lambda^2 + 14} + \frac{54\lambda}{9\lambda^2 + 14}\mathrm{i}$ or other similar expressions being common answers.

For part (b) the majority of students were able to score the first marks as long as they had an answer in terms of λ for part (a). Most used the tangent approach, with relative few realising they could just equate real and imaginary parts. However, nearly all lost the final mark as they did not reject the solution with negative imaginary part, which was argument $\frac{5\pi}{4}$, not $\frac{\pi}{4}$. Where students were unsuccessful in scoring the first mark in part (b), this was usually due to a failure to solve a quadratic λ either due to errors in (a) meaning a quadratic was not found, or through multiplying, rather than dividing, real and imaginary terms, while some students put the argument equal to $\frac{\pi}{4}$ rather than its tangent. There were a few students who demonstrated an awareness of the factor that the components both needed to be positive, but who, unfortunately, had made errors earlier in the question and so were not able to score the final mark despite evidencing a correct understanding of the requirements.

Question 10

Although proof by induction is another expected topic, it is one that consistently proves a challenge to students at lower grades, though there were very many excellent attempts from the higher grade students. Indeed, the overall performance compared to previous series was very strong on this question. Most demonstrated an understanding of the general procedure of verifying the base case, carrying out the inductive step and then concluding, but many had difficulties with the details of the inductive step. Conclusions were not always clear in the principles of "true for $n = 1$" and "true for $n = k$ implies true for $n = k + 1$", particularly the latter with statements of "true for $n = k$ and $n = k + 1$" or similar being common. Only a very few omitted a statement about being true for all *n* at the end.

Part (i) was the more approachable of the two parts overall. Many students failed to put $n = 1$ clearly into the both sides, particularly the right hand side needing clear demonstration, thus losing out on a very easy mark. However, most realised a check for $n = 1$ was necessary and a few were able to pick up all the remaining marks for otherwise correct work, lacking just a sufficient check of the base case. Most students correctly attempted the step from k to $k + 1$, by setting up a suitable matrix product, but a number made arithmetic errors either when expanding or during simplification. It was clear that most could identify the matrix they were aiming for, making the $k + 1$'s clear, as many were able to reach this despite earlier errors, but these could not recover the marks. Others tried bridging the gap between where they knew they needed to start and end, but details were sometimes muddy in such cases and careful checking of the matrices was required. A few incorrect attempts at adding rather than subtracting matrices were seen, and it was also fairly common for students to simply write out the matrix with $k + 1$ in place and think that was sufficient, but doing no show that it arises from the correct product of k-th power matrix with the original.

Part (ii) was more challenging, though those who had learned well the method scored very well. Again, a significant number of students failed to adequately verify the base case, but stopped at finding 518 without showing it is a multiple of 7. For the inductive step, there was some variety in the exact approach taken. Perhaps narrowly the most common method was to attempt $f(k+1)$ directly, but a similar number worked with $f(k+1) – f(k)$ instead while a smaller number with $f(k+1) – Mf(k)$ for $M \neq 0,1$. Though many knew to start with one of these, not all showed a clear indication of where they were heading, and stopped at a simplified expression but without $f(k)$ clearly identified within their right hand side, thus losing the dM mark and final accuracy marks. Those who did realise they needed to extract this usually did so well and achieved a suitable expression for the first A mark, but those working with a form $f(k+1) – Mf(k)$ often did not complete reasoning as to why the divisibility of $f(k)$ by 7 followed, losing the final mark irregardless of their conclusion. A few students, however, did spot that when $M = 1$ is used the right hand side becomes divisible by 7 without the need to identify $f(k)$ at all, and were able to progress, usually successful, to the end using this. Only a few set up an equation $f(k) = 7M$ or similar, and used this to demonstrate the divisibility of $f(k + 1)$ by 7.